1. Find the first four terms of the binomial expansion of $\sqrt[3]{1-2x}$. State the set of values of x for which the expansion is valid.

[6]

2.

i. Express $\dfrac{x}{(1+x)(1-2x)}$ in partial fractions.

[3]

ii. Hence use binomial expansions to show that $\dfrac{x}{(1+x)(1-2x)} = ax + bx^2 + \ldots,$ where a and b are constants to be determined.

State the set of values of x for which the expansion is valid.

[5]

3. Find the first three terms in the binomial expansion of $(4+x)^{\frac{3}{2}}$. State the set of values of x for which the expansion is valid.

[5]

4.

i. Find the first three terms of the binomial expansion of $\dfrac{1}{\sqrt[3]{1-2x}}$. State the set of values of x for which the expansion is valid.

[5]

ii. Hence find a and b such that $\dfrac{1-3x}{\sqrt[3]{1-2x}} = 1 + ax + bx^2 + \ldots$.

[3]

5. Given that $\left(1+\dfrac{x}{p}\right)^q = 1 - x + \dfrac{3}{4}x^2 + \ldots,$ find p and q, and state the set of values of x for which the expansion is valid.

[7]

6.

Find the first four terms of the binomial expansion of $(1-2x)^{\frac{1}{2}}$. [4]

State the set of values of x for which the expansion is valid.

7.

(a) Use the binomial expansion to show that $(1-2x)^{-\frac{1}{2}} \approx 1+x+\frac{3}{2}x^2$ for sufficiently small values of x. [2]

(b) For what values of x is the expansion valid? [1]

(c) Find the expansion of $\sqrt{\frac{1+2x}{1-2x}}$ in ascending powers of x as far as the term in x^2. [3]

(d) Use $x = \frac{1}{20}$ in your answer to part **(c)** to find an approximate value for $\sqrt{11}$. [2]

8.

(i) Express $\frac{5-x}{(2-x)(1+x)}$ in partial fractions. [3]

(ii) Hence or otherwise find the first 3 terms of the binomial expansion of $\frac{5-x}{(2-x)(1+x)}$ in ascending powers of x. [5]

9. **In this question you must show detailed reasoning.**

Given that

$$(1 + ax)^n = 1 + 6x - 6x^2 + \ldots,$$

where a and n are constants, find the values of a and n. [6]

10. (See Insert for Practice2 64003.)

(a) Show how the substitution $x = y - \frac{m}{3y}$ can be used to transform $x^3 + mx = n$ into a quadratic equation in y^3. [3]

(b) Show that, when $m > 0$, the resulting quadratic equation in y^3 has distinct real roots. [2]

11. (a) Find the first four terms in the expansion of $\left(1+\frac{x}{2}\right)^{-2}$. [3]

(b) State the range of values of x for which this expansion is valid. [1]

12. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1 + 3x)^{-1}$ [3]

(b) State the range of values of x for which this expansion is valid. [1]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Guidance
1			$\sqrt[3]{1-2x} = (1-2x)^{1/3}$	B1	n = 1/3 only. Do not MR for $n \neq 1/3$
			$= 1 + \frac{1}{3}(-2x) + \frac{\frac{1}{3}(-\frac{2}{3})}{2!}(-2x)^2 + \frac{\frac{1}{3}(-\frac{2}{3})(-\frac{5}{3})}{3!}(-2x)^3 + \ldots$	M1	all four **correct unsimplified** binomial coeffs (not nCr) soi
					condone absence of brackets only if it is clear from subsequent work that they were assumed
				B1	$1 - \frac{2}{3}x$ www in this term
			$= 1 - \frac{2}{3}x - \frac{4}{9}x^2 - \frac{40}{81}x^3 + \ldots$	B1	$\ldots\ -\frac{4}{9}x^2$ www in this term (not if used $2x$ for $(-2x)$ throughout)
				B1	$\ldots - \frac{40}{81}x^3$ www in this term If there is an error in say the third coeff of the expansion then M0 B1B0B1 is possible.
			Valid for $-½ < x < ½$ or $\lvert x\rvert < ½$	B1	Independent of expansion Allow ≤'s (valid in this case) or a combination. Condone also, say, $-½ < \lvert x\rvert < ½$ but not $x < ½$ or $-1 < 2x < 1$ or $-½ > x > ½$ **Examiner's Comments** The method for finding the binomial expansion was understood by almost all candidates. Many candidates scored full marks here. The most common errors were sign errors, the omission of the validity or the use of $2x$ throughout instead of $(-2x)$.
			Total	6	

2		i	$\frac{x}{(1+x)(1-2x)} = \frac{A}{1+x} + \frac{B}{1-2x}$		
		i	$\Rightarrow x = A(1 - 2x) + B(1 + x)$	M1	expressing in partial fractions of correct form (at any stage) and attempting to use cover up, substitution or equating coefficients Condone a single sign error for M1 only.
		i	$x = ½ \Rightarrow ½ = B(1 + ½) \Rightarrow B = 1/3$	A1	www cao
		i	$x = -1 \Rightarrow -1 = 3A \Rightarrow A = -1/3$	A1	www cao (accept A/(1 + x) + B/(1 − 2x), A = −1/3, B = 1/3 as sufficient for full marks without needing to reassemble fractions with numerical numerators)
		i			<u>Examiner's Comments</u> Whilst almost all candidates knew the general method for expressing the given fraction in partial fractions, there were a surprising number of numerical errors.
		ii	$\frac{x}{(1+x)(1-2x)} = \frac{-1/3}{1+x} + \frac{1/3}{1-2x}$		
		ii	$= \frac{1}{3}\left[(1-2x)^{-1} - (1+x)^{-1}\right]$ $= \frac{1}{3}[1 + (-1)(-2x) + \frac{(-1)(-2)}{2}(-2x)^2 + ... - (1 + (-1)x + \frac{(-1)(-2)}{2}x^2 + ...)]$	M1	correct binomial coefficients throughout for first three terms of **either** $(1 - 2x)^{-1}$ or $(1 + x)^{-1}$ oe i.e. 1, (−1), (−1)(−2)/2, not nCr form. Or correct simplified coefficients seen.
		ii	$= \frac{1}{3}[1 + 2x + 4x^2 + ... - (1 - x + x^2 + ...)]$	A1 A1	1 + 2x + 4x2 $1 - x + x^2$ (or 1/3/ −1/3 of each expression, ft their A/B) If $k(1 - x + x^2)$ (A1) not clearly stated separately, condone absence of inner brackets (i.e. $1 + 2x + 4x^2 - 1 - x + x^2$) **only if** subsequently it is clear that brackets were assumed, otherwise A1A0. [i.e. $-1 - x + x^2$ is A0 unless it is followed by the correct answer] Ignore any subsequent incorrect terms

		ii	$= \frac{1}{3}(3x + 3x^2 + ...) = x + x^2 + ...$ so $a = 1$ and $b = 1$	A1	or from expansion of $x(1 - 2x)^{-1}(1 + x)^{-1}$ www cao
		ii	OR $x(1 - x - 2x^2) = x(1 - (x + 2x^2))$ $= x(1 + x + 2x^2 + (-1)(-2)(x + 2x^2)^2/2 + \ldots\ldots)$	M1	correct binomial coefficients throughout for $(1 - (x + 2x^2))$ oe (i.e. 1, −1), at least as far as necessary terms $(1 + x)$ (NB third term of expansion unnecessary and can be ignored)
		ii	$= x(1 + x + 2x^2 + x^2 \ldots\ldots)$	A2	$x(1 + x)$ www
		ii	$= x + x^2 \ldots$ so $a = 1$ and $b = 1$	A1	www cao
		ii	Valid for $-½ < x < ½$ or $\|x\| < ½$	B1	independent of expansion. Must combine as one overall range. condone ≤ s (although incorrect) or a combination. Condone also, say $-½ < \|x\| < ½$ but not $x < ½$ or $-1 < 2x < 1$ or $-½ > x > ½$
		ii			<u>Examiner's Comments</u> Most candidates were able to use the binomial expansion correctly although there were sign errors - often from using (-2x) as (2x). The most common error-which was very common- was using $\frac{1}{3(1+x)} = 3(1+x)^{-1} = 3(1 - x + x^2 \ldots) = 3 - 3x + 3x^2$ and similarly for $\frac{1}{3(1-2x)}$. The other frequent error was in the validity. Some candidates omitted this completely but many others failed to combine the validities from the two expansions, or failed to choose the more restrictive option.
			Total	8	
3			$(4 + x)^{\frac{3}{2}} = 4^{\frac{3}{2}}(1 + \frac{1}{4}x)^{\frac{3}{2}}$	M1	dealing with the '4'to obtain $4^{3/2}(1 + \frac{x}{4})^{3/2}$

					(or expanding as $4^{3/2}+\frac{3}{2}4^{1/2}x+\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)4^{-1/2}\frac{x^2}{2!}+\ldots$ and having all the powers of 4 correct)
			$= 8(1+\frac{3}{2}(\frac{1}{4}x)+\frac{3}{2}.\frac{1}{2}.\frac{1}{2!}(\frac{1}{4}x)^2+\ldots$	M1	correct **binomial coeffs** for n = 3/2 ie 1, 3/2, 3/2.1/2.1/2! Not nCr form Indep of coeff of x Indep of first M1
			$= 8 + 3x$	A1	$8 + 3x$ www
			$+ 3/16\ x^2$	A1	… $+ 3/16\ x^2$ www Ignore subsequent terms
			Valid for $-4 < x < 4$ or $\lvert x\rvert < 4$	B1	accept ≤ s or a combination of < and ≤, but not $-4>x>4$, $\lvert x\rvert>4$, or say $-4<x$ condone $-4<\lvert x\rvert<4$ Indep of all other marks Allow MR throughout this question for $n = m/2$ where $m \in$ N, and m odd and then −1 MR provided it is at least as difficult as the original. **Examiner's Comments** Much here depended upon the candidate's ability to factorise correctly. On too many occasions the factor was found to be 4 or ¼ instead of 8. The general method for expanding the binomial expansion was understood and the binomial coefficients were usually correct. Some who had factorised correctly then forgot to include the 8 at the final stage. The validity was often correct but wassometimes omitted and a variety of incorrect responses were also seen including - $¼<x<¼$. Good candidates scored well in this question.
			Total	5	
4		i	$\frac{1}{\sqrt[3]{1-2x}} = (1-2x)^{-1/3}$	B1	$n = -1/3$. See below **SC** for those with $n = 1/3$

		i	$=1+(-\frac{1}{3})(-2x)+\frac{(-\frac{1}{3})(-\frac{4}{3})}{2!}(-2x)^2+...$	M1	All three **correct unsimplified binomial** coefficients (not nCr) soi condone absence of brackets only if it is clear from subsequent work that they were assumed
		i		B1	1 + (2/3)x + … www
		i	$=1+\frac{2}{3}x+\frac{8}{9}x^2+...$	B1	(8/9)x^2 www in this term If there is an error, in say, the third coefficient of the expansion then M0B1B0is possible **SC** For n = 1/3 award B1 for 1 − (2/3)x and B1 for −(4/9)x^2 (so max 2 out of the first 4 marks)
		i	Valid for $-\frac{1}{2}<x<\frac{1}{2}$ or $\lvert x\rvert<\frac{1}{2}$	B1	Independent of expansion. Accept, say, − 1/2 < $\lvert x\rvert$ < 1/2 or −1/2 ≤ x < 1/2 (must be strict inequality for + 1/2) **Examiner's Comments** The most common mistake in part (i) was to use a value of 2 rather than − 2 as the coefficient of x in each term of the expansion. The binomial coefficients were nearly always correct though a small number missed the 2! from the denominator of the x^2 term. While the majority of candidates used the correct value of n a small minority incorrectly used $n=\frac{1}{3}$ or $-\frac{2}{3}$. The range of validity of the expansion was done much better thanin previous years although the most common mistake was to give non-strict inequalities. Other mistakes included: • $-\frac{1}{2}<-x<\frac{1}{2}$ • $\lvert x\rvert>\frac{1}{2}$ • $\frac{1}{2}<x<-\frac{1}{2}$

					Examiner's Comments In part (ii) the majority of candidates correctly multiplied their answer from part (i) with (1–3*x*) and simplified this expression correctly to obtain the correct values of *a* and *b* . It was concerning, however, that a number of candidates wrote $(1-3x)(1+\frac{2}{3}x+\frac{8}{9}x^2+...)=1+\frac{2}{3}x+\frac{8}{9}x^2-3x-2x-\frac{8}{3}x^2+...$ or even more worryingly expanded $(1-3x)^1$ as $1-3x$ + higher order terms in *x*.
		ii	$\frac{1-3x}{\sqrt[3]{1-2x}}=(1-3x)(1+\frac{2}{3}x+\frac{8}{9}x^2+...)$		
		ii	$=1+\frac{2}{3}x+\frac{8}{9}x^2-3x-2x^2+...$	M1	Use of (1 − 3*x*) × their (1 + (2/3)*x* + (8/9)x^2 + …) and attempt at removal of brackets (condone absence of brackets but must have two terms in *x* and two terms in x^2)
		ii		A1ft	Correct simplified expansion following their expansion in (i). This mark is dependent on scoring both M marks in (i) and (ii)
		ii	$=1-\frac{7}{3}x-\frac{10}{9}x^2+...$	A1	cao or B3 www in either part **SC** following either M0 or M1, B1 for either a or b correct
			Total	8	
5			$\left(1+\frac{x}{p}\right)^q=1+q\frac{x}{p}+\frac{q(q-1)}{2!}\left(\frac{x}{p}\right)^2+\cdots$	M1*	One of $\frac{q}{p}x$ or $\frac{q(q-1)}{2!}\left(\frac{x}{p}\right)^2$ (soi), for example, $\frac{q}{p}=-1$ scores M1 A1
			$\frac{q}{p}=-1$ $\frac{q(q-1)}{2p^2}=\frac{3}{4}$	A1 A1	Allow *x*'s on both sides of equations (if correct)
			$q=-p\Rightarrow\frac{-p(-p-1)}{2p^2}=\frac{3}{4}$ or $\frac{q(q-1)}{2q^2}=\frac{3}{4}$	M1dep*	Eliminating *p* (or *q*) from simultaneous equations (not involving *x*) involving both variables oe – if M1A1A1 awarded followed by either *p* or *q* correct (www) this implies this M mark
			⇒ $p = 2$	A1	$p = 2$ www (or $q = -2$)

			$\Rightarrow q = -2$	A1ft	$q = -2$ (or $p = 2$) for second value, ft their p or q (e.g. the negative of their p or q) provided first 4 marks awarded and only a single computational error in the method – so must be a correct method for solving their equation in p or q (ignore mention of p and/or $q = 0$)
			Valid for $\left\|\frac{x}{2}\right\| < 1 \Rightarrow \|x\| < 2$	A1	or $-2 < x < 2$ www, allow $-2 < \|x\| < 2$ but not say, $x < 2$ **SC If M0 M0 awarded and no wrong working seen then B1 for $p = 2$ and $q = -2$, B1 for $-2 < x < 2$ (oe) so max 2 marks** Guidance for solving quadratics on this paper: use of correct quadratic equation formula (if formula is quoted correctly then only one sign slip is permitted, if the formula is quoted incorrectly M0, if not quoted at all substitution must be completely correct to earn the M1) or factorising (giving their x^2 term and one other term when factors multiplied out) or completing the square (must get to the square root stage involving ± and arithmetical errors may be condoned provided that perfect square term was correct) **Examiner's Comments** The binomial expansion of $\left(1+\frac{x}{p}\right)^q$ was done extremely well by the vast majority of candidates with the most common error being the failure to correctly deal with the x^2 term with many giving the coefficient (of this term) as $\frac{q(q-1)}{2p}$ rather than the correct $\frac{q(q-1)}{2p^2}$. It was surprising how few candidates could go on to form the correct pair of simultaneous equations and fewer still who could solve this pair of equations accurately and successfully. Those candidates who correctly found the value of p usually went on to state the set of values of x for which the expansion was valid.
			Total	7	

6			$\approx 1+\frac{1}{2}(-2x)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}(-2x)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-2x)^3$ $=1-x-\frac{1}{2}x^2-\frac{1}{2}x^3$ valid for $-\frac{1}{2}<x<\frac{1}{2}$	M1(AO1.1) A2(AO1.1 1.1) B1(AO2.3) [4]	binomial coefficients seen, allow one error $1-x$, $-\frac{1}{2}x^2$, $-\frac{1}{2}x^3$ or A1 for 2 correct terms or $\lvert x\rvert<\frac{1}{2}$	In this case, the series converges for $x=\pm\frac{1}{2}$ candidates are not expected to know this but allow ≤ for either or both inequalities.
			Total	4		

7		a	$(1-2x)^{-\frac{1}{2}} \approx 1+\left(-\frac{1}{2}\right)(-2x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(-2x)^2+\ldots$ $=1+x+\frac{3}{2}x^2$ **AG**	M1(AO 1.1b) A1(AO 2.1) [2]	Correct form required; allow sign errors Must be correctly obtained
		b	Valid when $\lvert x \rvert < \frac{1}{2}$	E1(AO 2.3) [1]	
		c	$(1+2x)^{\frac{1}{2}}=1+x-\frac{1}{2}x^2+\ldots$ $\left(1+x-\frac{1}{2}x^2\right)\left(1+x+\frac{3}{2}x^2\right)$ $= 1 + 2x + 2x^2$ Alternative method $\sqrt{\frac{1+2x}{1-2x}}=\sqrt{\frac{(1+2x)(1+2x)}{(1-2x)(1+2x)}}=\frac{1+2x}{\sqrt{1-4x^2}}$ $=(1+2x)\left(1+\left(-\frac{1}{2}\right)(-4x^2)+\ldots\right)$ $= 1 + 2x + 2x^2$	B1(AO 1.1a) M1(AO 1.1a) A1(AO 1.1b) M1 M1 A1	Product of their expansions attempted Converting to rational numerator form Expand denominator and multiply out

				[3]	
		d	$\sqrt{\frac{1.1}{0.9}} = \frac{\sqrt{11}}{3} \approx 1 + 2\times\frac{1}{20} + 2\times\left(\frac{1}{20}\right)^2$ $\sqrt{11} \approx 3.315$	M1(AO 2.1) A1(AO 2.2a) [2]	Obtaining an expression involving $\sqrt{11}$ Or $\frac{663}{200}$
			Total	8	
8		i	$\frac{5-x}{(2-x)(1+x)} = \frac{A}{2-x} + \frac{B}{1+x}$ $\Rightarrow 5 - x = A(1 + x) + B(2 - x)$ $x = 2 \Rightarrow 3 = 3A,\ A = 1$ $x = -1 \Rightarrow 6 = 3B \Rightarrow B = 2$	M1 A1 A1 [3]	Cover up, substitution or equating coefficients **Examiner's Comments** Part (i) was answered extremely well with the vast majority of candidates correctly expressing $\frac{5-x}{(2-x)(1+x)}$ n partial fractions.
		ii	$\frac{A}{2-x} = \frac{A}{2}(1-\frac{1}{2}x)^{-1}$ $= \frac{A}{2}(1+(-1)(-\frac{1}{2}x)+\frac{(-1)(-2)}{2!}(-\frac{1}{2}x)^2+...)$	B1 M1	Or equivalent All three correct **unsimplified** binomial coefficients (not nCr) soi **for either expansion** i.e. 1, −1 and $\frac{(-1)(-2)}{2}$. Or correct simplified coefficients seen Ignore any subsequent incorrect terms – ft their *A* from (i) only

			$= A\left(\frac{1}{2}+\frac{1}{4}x+\frac{1}{8}x^2+\ldots\right)$ $\frac{B}{1+x} = B(1+x)^{-1} = B(1-x+x^2+\ldots)$ $\frac{5-x}{(2-x)(1+x)} = \frac{5}{2}-\frac{7}{4}x+\frac{17}{8}x^2+\ldots$	A1ft A1ft A1 [5]	Ignore any subsequent incorrect terms – ft their *B* from (i) only www cao – ignore any higher order terms stated – isw after correct expansion seen **Examiner's Comments** In part (ii) most candidates used their answer to part (i) in their attempt to find the binomial expansion of $\frac{5-x}{(2-x)(1+x)}$ although some candidates did (with varying degrees of success) attempt to expand $(5 - x)$ $(2 - x)(1 + x)^{-1}$ directly. Whilst the majority of candidates correctly dealt with the expansion of $\frac{2}{1+x}$ (and so scored at least two marks in this part) it was surprising how many candidates (at this level) struggled in re-writing $\frac{1}{2-x}$ as $\frac{1}{2}\left(1-\frac{1}{2}x\right)^{-1}$. In some cases it was clear that candidates either did not realise or even recognise that the 2 inside the bracket had to be removed before this term could be binomially expanded. Those candidates who expanded both terms correctly usually went on to score full marks.
			Total	8	
9			$na = 6$ $\frac{n(n-1)}{2!}a^2 = -6$ Substitution of $a = \frac{6}{n}$ in second equation oe $18(n - 1) = -6n$ soi	M1(AO3.1a) A1(AO2.1) M1(AO1.1b) A1(AO1.1b) A1(AO1.1b) A1(AO1.1b)	

			$n = \frac{3}{4}$ $a = 8$	[6]	
			Total	6	
10		a	$x^3 = \left(y - \frac{m}{3y}\right)^3$ $= y^3 - 3y^2\left(\frac{m}{3y}\right) + 3y\left(\frac{m}{3y}\right)^2 - \left(\frac{m}{3y}\right)^3$ $= y^3 - my + \frac{m^2}{3y} - \frac{m^3}{27y^3}$ $x^3 + mx = y^3 - \frac{m^3}{27y^3}$ $y^3 - \frac{m^3}{27y^3} = n \Rightarrow \left(y^3\right)^2 - \frac{m^3}{27} = ny^3$ This is a quadratic in y^3	M1(AO1.1a) A1(AO1.1) E1(AO2.1) [3]	Successful completion
		b	$\left(y^3\right)^2 - ny^3 - \frac{m^3}{27} = 0$ has distinct real roots when $(-n)^2 - 4\left(-\frac{m^3}{27}\right) > 0$	M1(AO3.1a) E1(AO2.1)	Use of their $b^2 - 4ac$

			$n^2 + \frac{4m^3}{27} > 0$ — certainly true when $m > 0$	[2]	
			Total	5	
11		a	$1+(-2)\frac{x}{2}+\frac{(-2)(-3)}{2!}\left(\frac{x}{2}\right)^2+\frac{(-2)(-3)(-4)}{3!}\left(\frac{x}{2}\right)^3$ $1-x+\frac{3}{4}x^2-\frac{1}{2}x^3$	M1 (AO1.1a) A1 (AO1.1) A1 (AO1.1) [3]	allow sign errors and one coefficient error three out of four terms correct all four terms correct ignore extra terms
		b	$-2 < x < 2$ oe	B1 (AO2.3) [1]	
			Total	4	
12		a	$1+(-1)(3x)+(-1)(-2)\frac{(3x)^2}{2!}+(-1)(-2)(-3)\frac{(3x)^3}{3!}$ $1-3x+9x^2-27x^3$	M1(AO 1.1) A1(AO 1.1) A1(AO 1.1) [3]	allow sign errors allow recovery from omission of brackets
		b	$\lvert x \rvert < ⅓$	B1(AO 1.1) [1]	$-⅓ < x < ⅓$
			Total	4	